



GN-232

V Semester B.A./B.Sc. Examination, December - 2019
(CBCS) (F+R) 2016-17 and Onwards)

MATHEMATICS - V

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** questions.

PART - A

Answer **any five** questions.

5x2=10

1. (a) Give an example of
 - (i) a ring with zero divisor
 - (ii) a non-commutative ring with unity
- (b) In a ring $(R, +, \cdot)$ prove that $a \cdot (b - c) = a \cdot b - a \cdot c \forall a, b, c \in R$.
- (c) Define principal and maximal ideals of a ring R .
- (d) Find the maximum directional derivative of $\phi = x^3 y^2 z$ at the point $(1, -2, 3)$.
- (e) If $\vec{f} = 3x^2 \hat{i} + 5xy^2 \hat{j} + xyz^3 \hat{k}$ then, find $\text{div } \vec{f}$ at $(1, 2, 3)$.
- (f) Evaluate : $\Delta^4(1 - ax)(1 - bx)(1 - cx)(1 - dx)$.
- (g) Write Lagrange's Interpolation formula for unequal intervals.
- (h) Using Trapezoidal rule, evaluate $\int_0^6 f(x) dx$ given :

x	0	1	2	3	4	5	6
$f(x)$	0.146	0.161	0.176	0.190	0.204	0.217	0.230

PART - B

Answer **two** full questions.

2x10=20

2. (a) Prove that the set $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring w.r.t. \oplus_6 and \otimes_6 as two compositions.
- (b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in it.

OR

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3. (a) Prove that the necessary and sufficient conditions for a non-empty subset S to be a subring of R , are :
- (i) $S + (-S) = S$ (ii) $SS \subseteq S$
- (b) Define the right and left ideals of a ring R . Show that the subset
- $$S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \mid a, b \in Z \right\} \text{ of } M_2(Z) \text{ is a left ideal but not a right ideals of } M_2(Z).$$

4. (a) (i) If 'a' is an element of a commutative ring R , then prove that $aR = \{ar \mid r \in R\}$ is an ideal of 'R'.
- (ii) If I is an ideal of a ring 'R' with unity and $1 \in I$ then prove that $I = R$.
- (b) Find all the principal ideals of the ring $R = \{0, 1, 2, 3, 4, 5\}$ w.r.t. \oplus_6 and \otimes_6 as two compositions.

OR

5. (a) If $f: R \rightarrow R'$ is a homomorphism of a ring R into R' then prove that
- (i) $f(0) = 0'$ where 0 and $0'$ are the zero elements of R and R' respectively.
- (ii) $f(-a) = -f(a) \forall a \in R$.
- (b) State and prove fundamental theorem of homomorphism of rings.

PART - C

Answer **two** full questions.

2x10=20

6. (a) Find the constants a and b so that the surfaces $x^2 + ayz = 3x$ and $bx^2y + z^3 = (b-8)y$ intersect orthogonally at the point $(1, 1, -2)$.
- (b) If ϕ is a scalar point function and \vec{f} is a vector point function then prove that $\text{div}(\phi \vec{f}) = \phi(\text{div} \vec{f}) + (\text{grad} \phi) \cdot \vec{f}$

OR

7. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then prove that $\nabla^2(r^3 \vec{r}) = 18r \vec{r}$ where $r = |\vec{r}|$

- (b) If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ then find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$

8. (a) Show that $\text{div}(\vec{a} \times (\vec{r} \times \vec{a})) = 2|\vec{a}|^2$ where \vec{a} is a constant vector.

- (b) Show that $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla \phi$

OR



9. (a) (i) If $\vec{F} = 3xy\hat{i} + 20yz^2\hat{j} - 15xz\hat{k}$ and $\phi = xyz$, then find $\text{curl}(\phi\vec{F})$.
- (ii) Show that $\vec{F} = 2x^2z\hat{i} - 10xyz\hat{j} + 3xz^2\hat{k}$ is solenoidal.
- (b) Find $\text{curl}(\text{curl}\vec{F})$ if $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

PART - D

Answer **two** full questions.**2x10=20**

10. (a) Use the method of separation of symbols to prove that

$$u_0 - u_1 + u_2 - u_3 + \dots + \infty = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \frac{1}{16}\Delta^3 u_0 + \dots$$

- (b) Obtain a function whose first difference is $x^3 + 3x^2 + 5x + 12$.

OR

11. (a) Find the number of students from the following data who secured marks not more than 45.

Marks	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of Students	35	48	70	40	22

- (b) Find a polynomial of lowest degree which assumes the values 10, 4, 40, 424, 620 at $x = -2, 1, 3, 7$ and 8 respectively, using Newton's divided difference formula.

12. (a) By employing Newton-Gregory backward difference formula, find $f(9.7)$ from the following data.

x	8	8.5	9	9.5	10
$f(x)$	50	57	64	71	75

- (b) Using Simpson's $\frac{1}{3}$ rule, Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ dividing the interval (0, 1) into 8 equal parts.

OR

13. (a) Applying Lagrange's formula find $f(5)$, given that $f(1)=2, f(2)=4, f(3)=8$ and $f(7)=128$.

- (b) Using Simpson's $\frac{3}{8}$ rule, Evaluate $\int_4^{5.2} \log_e x dx$ taking $h=0.2$.

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